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# Exact solution of the O(2) symmetric Thirring model with dynamical generation of a superconducting gap

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**Abstract.** We present, for $N = 2$, an exact operator solution for the O($N$) symmetric Thirring model and show that it undergoes dynamical generation of a superconducting gap.

## 1. Introduction

One of the nice features of quantum field theories formulated in two-dimensional spacetime is the fact that many of them may be exactly solved [1]. One is thereby allowed to have non-trivial information about the effective dynamical behaviour and actual physical content of the theory. In recent years, the discovery of many applications of two-dimensional field theory models in realistic systems of condensed matter physics, on the other hand, showed that these theories were more than just toy models or theoretical laboratories, but genuine theories, capable of making experimental predictions [2]. The combination of these two aspects enhances the interest in the study of models which are potentialy relevant from the application point of view and which may be exactly solved.

In a recent publication, the O($N$) symmetric Thirring model containing $N$ species (colours) of fermions $\psi_a$, $a = 1, 2, \ldots N$, was analysed [3]. The Lagrangian density of this model is given by

$$\mathcal{L} = \mathrm{i}\,\overline{\psi}_a\, \gamma_\mu \partial^\mu\, \psi_a - \tfrac{1}{2} g J_{\mu ab}\, J^\mu_{ab} \tag{1.1}$$

which can be written in terms of the Dirac components as

$$\mathcal{L} = \mathrm{i}\,\psi^*_{(1)a}\partial_-\psi_{(1)a} + \mathrm{i}\,\psi^*_{(2)a}\partial_+\psi_{(2)a} - 2g\; \psi^*_{(2)a}\psi^*_{(1)a}\psi_{(1)b}\psi_{(2)b} \tag{1.2}$$

where sum over $a, b$ is understood, $J^\mu_{ab} = \overline{\psi}_a\, \gamma^\mu\, \psi_b$, $\partial_\pm = \partial_0 \pm \partial_1$. It was shown that, in the large-$N$ limit, this theory presented dynamical generation of a superconducting gap [3]. It was conjectured also [3], that this mechanism was exact for all values of $N$, as in the analogous mass generation of the chiral Gross–Neveu model [4] and that the model could, perhaps, be exactly solved. In a subsequent paper [5], an extended version of the model was shown to have two phases, one presenting dynamical superconducting gap generation, and another presenting dynamical mass generation; the two mechanisms are competitive for duality reasons [5].

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In this paper, we consider the $N = 2$ case of the model described by (1.1) and show its equivalence with the exactly solved massive Thirring model (MTM). In [6] we showed that the MTM is equivalent to the superconducting Thirring model (SCTM).

The model described by (1.1) and (1.2) is a special case of the one considered in [7], but this special case lies outside the condition imposed on the coupling constant in [7] to show integrability.

In addition to the U(1) and chiral U(1) symmetries, the interaction part of (1.1) possesses complex $O_c(N)$ invariance and the kinetic part SU($N$) invariance. Since the intersection of these two groups is real O($N$), the full symmetry group is (real O($N$))$\otimes$ U(1)$\otimes$ chiral U(1). After introducing an Abelian bosonisation well suited to the O(2) group, we show that the U(1) and chiral U(1) symmetries decouple from the physical sector implying, in particular, that the physical superconducting gap operator $\tilde{\sigma}$ will create uncharged U(1) 'Cooper pairs'. We show that this physical SC gap operator is fermionic and therefore may not condense. The bosonic composite operator $\tilde{\sigma}\tilde{\sigma}$, however, does acquire a non-zero vacuum expectation value. The model therefore generates a superconducting gap without breaking the U(1) symmetry, in agreement with the Coleman–Mermin–Wagner theorem [8, 9]. As we will see, the spectrum consists of an infinite number of massive particles which are O(2) charge neutral. The exact solution of the model follows from the well known [9–11] connection between the exactly solved massive (superconducting) Thirring model and the sine-Gordon theory. The O(2) charge of our model appears as the U(1) charge of the massive Thirring model.

## 2. Bosonisation of the theory

In this section we use Abelian bosonisation to map the O(2) theory into an exactly soluble model, namely the massive (superconducting) Thirring model [6].

Let us start by introducing the fields $\chi$ and $\theta$, by

$$\chi = \frac{1}{\sqrt{2}}\,(\psi_1 + i\psi_2) \qquad \theta = \frac{1}{\sqrt{2}}\,(\psi_1 - i\psi_2) \tag{2.1}$$

where 1 and 2 are O(2) indices. The fermion current is given by

$$J_a^\mu = \overline{\psi}_a\,\gamma^\mu\psi_a = \overline{\chi}\gamma^\mu\chi + \overline{\theta}\,\gamma^\mu\theta + q_a(\overline{\chi}\gamma^\mu\theta + \overline{\theta}\gamma^\mu\chi) \tag{2.2}$$

$$J^5_{\mu a} = \varepsilon_{\mu\nu}\,J^\nu_a \tag{2.3}$$

where $q_a = 1, -1$ when $a = 1, 2$. The interacting current $J^\mu_{ab}$ appearing in the Lagrangian density (1.1) is given by (2.2) for $a = b$ and by

$$J^\mu_{ab} = \overline{\psi}_a\gamma^\mu\psi_b = \frac{1}{2i}[\varepsilon_{ab}(\overline{\chi}\gamma^\mu\chi - \overline{\theta}\gamma^\mu\theta) - \overline{\chi}\gamma^\mu\theta + \overline{\theta}\gamma^\mu\chi] \qquad a \neq b \tag{2.4}$$

for $a \neq b$. The conserved U(1), chiral U(1) and O(2) currents are given by

$$J^\mu = \sum_{a=1}^{2} J^\mu_a = 2(\overline{\chi}\gamma^\mu\chi + \overline{\theta}\gamma^\mu\theta) \tag{2.5}$$

$$J^5_\mu = \varepsilon_{\mu\nu}\,J^\nu \tag{2.6}$$

$$J_\mu^{O(2)} = \sum_{a,b=1}^{2} i\varepsilon_{ab}\, \overline{\Psi}_a \gamma_\mu \Psi_b = (\overline{\chi}\, \gamma_\mu \chi - \overline{\theta}\, \gamma_\mu\, \theta). \tag{2.7}$$

We introduce the Mandelstam representation for the fermion fields $\chi$ and $\theta$ [12]

$$\chi(x) = \left(\frac{m}{2\pi}\right)^{1/2} \exp\left(-i\frac{\pi}{4}\gamma^5\right) \,:\, \exp\left[i\sqrt{\pi}\left(\gamma^5\widetilde{\phi}_1(x) + \eta_1(x)\right)\right] \,: \tag{2.8}$$

$$\theta(x) = \left(\frac{m}{2\pi}\right)^{1/2} \exp\left(-i\frac{\pi}{4}\gamma^5\right) \,:\, \exp\left[i\sqrt{\pi}\left(\gamma^5\widetilde{\phi}_2(x) + \eta_2(x)\right)\right] \,: \tag{2.9}$$

where

$$\eta_a(x) = \int_{x_1}^{\infty} \partial_{x_0}\, \widetilde{\phi}_a(x_0, z_1)\mathrm{d}z_1. \tag{2.10}$$

Introducing the two independent scalar fields

$$\widetilde{\phi}^{\pm} = \frac{1}{\sqrt{2}}\,(\widetilde{\phi}_1 \pm \widetilde{\phi}_2) \tag{2.11}$$

which satisfy canonical commutation relations at short distance, and using the point-splitting limit prescription we obtain the conserved currents

$$J^\mu = -\sqrt{\frac{2}{\pi}}\, \varepsilon_{\mu\nu}\, \partial_\nu\, \widetilde{\phi}^{+} \tag{2.12}$$

$$J_\mu^{O(2)} = -i\sqrt{\frac{2}{\pi}}\, \varepsilon_{\mu\nu}\, \partial^\nu\, \widetilde{\phi}^{-}. \tag{2.13}$$

As we will see, the field $\widetilde{\phi}^{+}$ is free and massless

$$\varepsilon_{\mu\nu}\, \partial^\nu\, \widetilde{\phi}^{+} = \partial_\mu\, \phi^{+}(x) \tag{2.14}$$

guaranteeing, therefore U(1) and chiral U(1) conservation at the quantum level.

The original O(2) transformation for the $\psi_a$ fields, i.e.

$$\psi_1' = -\psi_1\, \cos\Omega + \psi_2\, \sin\Omega \qquad \psi_2' = \psi_1\, \sin\Omega + \psi_2\, \cos\Omega \tag{2.15}$$

acts on the $\chi$ and $\theta$ fields as

$$\chi' = e^{-i\Omega}\, \chi \qquad \theta' = e^{i\Omega}\, \theta \tag{2.16}$$

which corresponds to a U(1) charge transformation (gauge transformation of the first kind) implemented by the unitary operator

$$U = \exp(i\Omega Q^{O(2)}) \tag{2.17}$$

where the O(2) generator is given by

$$Q^{O(2)} = \int_{-\infty}^{+\infty} J_0^{O(2)}(x)\mathrm{d}x_1. \tag{2.18}$$

From (2.16) we see that $\chi$ and $\theta$ have opposite U(1) (O(2)) charges. Writing the fermion fields $\chi$ and $\theta$ in terms of the $\phi^{\pm}$ fields, we see that they factorise as

$$\chi = \psi_0 \widehat{\chi} \tag{2.19}$$

$$\theta = \psi_0 \widehat{\theta} \tag{2.20}$$

where $\psi_0$ is a free massless non-canonical field with spin $S = \frac{1}{4}$, given by (see 2.14)

$$\psi_0(x) = :\exp\left[\mathrm{i}\,(\pi/2)^{1/2}\left\{\gamma^5\widetilde{\phi}^+(x) + \phi^+(x)\right\}\right]: . \tag{2.21}$$

The physical content of the theory, however, must be solely in the interacting fields $\widehat{\chi}$ and $\widehat{\theta}$ which are given by

$$\widehat{\chi} = \left(\frac{m}{2\pi}\right)^{1/2} \exp\left(-\mathrm{i}\,\frac{\pi}{4}\gamma^5\right) :\exp\left[\mathrm{i}\,(\pi/2)^{1/2}\left\{\gamma^5\widetilde{\phi}^-(x) + \eta^-(x)\right\}\right]: \tag{2.22}$$

$$\widehat{\theta} = \exp\left(-\mathrm{i}\,\frac{\pi}{2}\gamma^5\right)\widehat{\chi}^*(x) \tag{2.23}$$

which satisfy neither Fermi nor Bose statistics, but have spin-$\frac{1}{4}$. Equation (2.23) means that the two 'physical' interacting objects $\widehat{\chi}$ and $\widehat{\theta}$ are not independent and the physical content of the theory can be described in terms of just one of them. This property is the analogue for $N = 2$ of the larger-$N$ feature that antiparticles are bound states of $N - 1$ particles in similar systems [13]. Using (2.22) and (2.25), the O(2) current (2.7), (2.13) may be written in terms of $\widehat{\chi}$ fields

$$\mathrm{i}J_\mu^{\mathrm{O}(2)} = -(2/\pi)^{1/2}\,\varepsilon_{\mu\nu}\,\partial^\nu\,\widetilde{\phi}^- = 2\bar{\widehat{\chi}}\gamma_\mu\widehat{\chi}. \tag{2.24}$$

One can introduce the superconducting order parameter $\sigma$ given by the composite operator

$$\sigma = \sum_{a=1}^{2} \vdots\psi_{(1)a}\,\psi_{(2)a}\vdots = \vdots\chi_{(1)}\,\theta_{(2)}\vdots + \vdots\theta_{(1)}\,\chi_{(2)}\vdots \tag{2.25}$$

where the notation $\vdots\ \vdots$ means normal ordering with respect to the fermion field operator and the numbers between brackets are Dirac indices. Using the decomposition (2.19), (2.20) and (2.23), we find

$$\sigma = \vdots\psi_{0(1)}\,\psi_{0(2)}\vdots(\vdots\widehat{\chi}^*_{(1)}\,\widehat{\chi}_{(2)}\vdots + \vdots\widehat{\chi}_{(1)}\,\widehat{\chi}^*_{(2)}\vdots) = \vdots\psi_{0(1)}\,\psi_{0(2)}\vdots\,\widehat{\sigma} \tag{2.26}$$

where

$$\widehat{\sigma} = \vdots\widehat{\chi}^*_{(1)}\widehat{\chi}_{(2)}\vdots + \vdots\widehat{\chi}_{(1)}\,\widehat{\chi}^*_{(2)}\vdots = \frac{m}{\pi}\ :\cos\left(\sqrt{2\pi}\widetilde{\phi}^-\right): \tag{2.27}$$

$$\vdots\psi_{0(1)}\,\psi_{0(2)}\vdots =: \exp(-\mathrm{i}\sqrt{2\pi}\,\eta^+(x)): \tag{2.28}$$

with $\eta^+$ given by (2.10) (with $\widetilde{\phi}_a$ replaced by $\widetilde{\phi}^+$) being the potential of the U(1) charge current and $\widehat{\sigma}$ is the physical superconducting order parameter with zero U(1) charge.

The Lagrangian density (1.1) can be written as

$$\mathscr{L} = \mathrm{i}\bar{\chi}\gamma^\mu \partial_\mu \chi + \mathrm{i}\bar{\theta}\gamma^\mu \partial_\mu \theta + 2g : \sigma\sigma^* : \tag{2.29}$$

which, in terms of the physical operators, is given by

$$\mathscr{L} = \tfrac{1}{2} : (\partial_\mu \widetilde{\phi}^+)^2 : + \mathrm{i}\bar{\widehat{\chi}}\gamma^\mu \partial_\mu \widehat{\chi} + \mathrm{i}\bar{\widehat{\theta}}\gamma^\mu \partial_\mu \widehat{\theta} + 2g : \widehat{\sigma}\widehat{\sigma} : . \tag{2.30}$$

As anticipated, $\widetilde{\phi}^+$ is a free massless field. The fully bosonised Lagrangian density is then given by

$$\alpha\mathscr{L} = \mathscr{L}_0^+ + \mathscr{L}^- \tag{2.31}$$

where

$$\alpha = \left(1 + \frac{2g}{\pi}\right)^{-1} \tag{2.32}$$

$\mathscr{L}_0^+$ is the free $\phi^+$ Lagrangian:

$$\mathscr{L}^- = \tfrac{1}{2} : (\partial_\mu \widetilde{\phi}^-)^2 : + G : \cos(\sqrt{8\pi}\,\widetilde{\phi}^-) : \tag{2.33}$$

and

$$G = \left(\frac{m^2}{2\pi}\right) \frac{2g/\pi}{1 + 2g/\pi}. \tag{2.34}$$

The physical content of the model is given by a sine-Gordon theory with $\beta^2 = 8\pi$.

From (2.26) and the fact that $\widehat{\chi}$ has spin-$\frac{1}{4}$, we see that the spin of $\widehat{\sigma}$ is $\frac{1}{2}$. The vacuum expectation value of $\widehat{\sigma}$ must, therefore, vanish since a fermionic excitation may not condense. This can also be seen from (2.27) and (2.33), considering that the coefficient of $\widetilde{\phi}^-$ in $\widehat{\sigma}$ is one half of the corresponding coefficient in $\mathscr{L}^-$. In a Coulomb gas description of the sine-Gordon theory [10, 11], this would mean the neutrality condition would never be attained for $\langle\widehat{\sigma}\rangle$, implying $\langle\widehat{\sigma}\rangle = 0$. The composite operator $\widehat{\sigma}\widehat{\sigma}$, however, has integer spin and may therefore condense. Again, in a Coulomb gas version of the theory one would find that $\widehat{\sigma}\widehat{\sigma}$ introduces an external charge whose modulus is identical to the moduli of the internal moduli ($\sqrt{8\pi}$). This implies that $\langle\widehat{\sigma}\widehat{\sigma}\rangle \neq 0$, which means the generation of a SC gap.

In a model with dynamical mass generation, as for example, the SU(2) chiral Gross–Neveu model [4, 13], the physical mass gap operator $\widehat{\pi}$ is obtained by the extraction of the 'decoupled' massless excitation carrying the chiral U(1) selection rule, i.e.

$$\pi \equiv \sum_{a=1}^{2} \vdots\psi^*_{(1)a}\, \psi_{(2)a}\vdots = \vdots\psi^*_{0(1)}\, \psi_{0(2)}\vdots\,\widehat{\pi} \tag{2.35}$$

with

$$\vdots\psi^*_{0(1)}\, \psi_{0(2)}\vdots = \exp(\mathrm{i}\sqrt{2\pi}\,\widetilde{\phi}^+) : \tag{2.36}$$

and

$$\widehat{\pi}(x) = \frac{m}{\pi} \ :\cos\left(\sqrt{2\pi}\ \widetilde{\phi}^-\right): \tag{2.37}$$

the chiral U(1) charge being given by

$$Q^5 = \int_{-\infty}^{+\infty} \dot{\widetilde{\phi}}^+ (x) \mathrm{d}x^1. \tag{2.38}$$

In the model under consideration the physical superconducting order parameter $\widehat{\sigma}$ is obtained by extracting the decoupled massless excitation carrying the U(1) charge selection rule, namely

$$\widehat{\sigma} =: \exp(-\mathrm{i}\sqrt{2\pi}\,\eta^+)\,\sigma :. \tag{2.39}$$

The generator of the U(1) transformations is given by

$$Q = \int_{-\infty}^{+\infty} \dot{\eta}^+(x)\mathrm{d}x^1 \tag{2.40}$$

and the fact that $\sigma$ does not commute with $Q$ prevents $\langle 0 \mid \sigma \mid 0 \rangle$ from developing a non-zero value. The physical operator $\widehat{\sigma}$, however, commutes with $Q$ and in this sense we may say that the model under consideration exhibits dynamical generation of a superconducting gap without the breakdown of the corresponding continuous U(1) charge symmetry. It is interesting to note that in the present O(2) symmetric theory the physical superconducting gap and mass operators $\widehat{\sigma}$ and $\widehat{\pi}$ are identical ($\sigma$ and $\pi$ are not!) and therefore do not satisfy a dual algebra as in the case of $N > 2$. This fact was already observed in [5]. The case $N = 2$ is quite unique because the O($N$) group is then Abelian. The bosonisation scheme used in the $N > 2$ cases [5] leads, when applied to $N = 2$, to a non-local O(2) current and the identification of the O(2) charge degrees of freedom in the final physical system is rather difficult. For $N > 2$ the conservation of the O($N$) currents is ensured by the equation of motion [5, 14].

## 3. Description in terms of a spin-$\frac{1}{2}$ Thirring field

It is convenient to introduce a spin-$\frac{1}{2}$ field $\widetilde{\psi}$ in terms of which the physical sector of the model may also be described. We define it as

$$\widetilde{\psi}(x) =: \exp[\mathrm{i}\,(\pi/2)^{1/2}\gamma^5\widetilde{\phi}^-(x)]\,\widetilde{\chi}(x) : \tag{3.1}$$

whereupon

$$\widetilde{\psi}(x) = \left(\frac{m}{2\pi}\right)^{1/2} \exp\left(-\mathrm{i}\,\frac{\pi}{4}\gamma^5\right) : \exp\{\mathrm{i}[\sqrt{2\pi}\,\gamma^5\,\widetilde{\phi}^-(x) + (\pi/2)^{1/2}\eta^-(x)]\} :. \tag{3.2}$$

We may relate the O(2) current, equation (2.24), with the U(1) current for $\widetilde{\psi}$:

$$J_\mu^{\mathrm{O}(2)} = -(2/\pi)^{1/2}\,\varepsilon_{\mu\nu}\,\partial^\nu\widetilde{\phi}^- = \,:\overline{\widetilde{\psi}}\,\gamma_\mu\,\widetilde{\psi}:\, = \widetilde{J}^\mu. \tag{3.3}$$

Inserting (3.2) in (2.30), we may express the interacting Lagrangian (2.3) in terms of the fermion field $\widetilde{\psi}$ as

$$\mathscr{L}^{-}(x) = \overline{\widetilde{\psi}}(x)\gamma_\mu\, \partial^\mu \widetilde{\psi} + (m/\pi)\, g\alpha {:}\overline{\widetilde{\psi}}(x)\widetilde{\psi}(x){:} - (\pi/2)\, \widetilde{J}_\mu(x)\, \widetilde{J}^\mu(x). \tag{3.4}$$

We see that the effective dynamics of the theory, as described in terms of this field, contains a Thirring interaction with coupling constant $g_{\mathrm{Th}} = -(\pi/2)$ [12, 15] and a mass gap.

From the above connection with the massive Thirring model, we conclude that the spectrum of the O(2) model and its $S$-matrix in terms of the $\widetilde{\psi}$ particles must coincide with those of the massive Thirring model at $\beta^2 = 8\pi$ ($g_{\mathrm{Th}} = -\pi/2$ and $M = (m/2)\ [(2g/(\pi + 2g)]$). This last relation follows from the ones between $\alpha$ and $M$ and $\alpha$ and $G$. Using the exact solution of the massive Thirring model at $g_{\mathrm{Th}} = -\pi/2$, which was obtained by Korepin [16], we conclude that the spectrum of our O(2) model consists of an infinite number of (O(2)) charge-neutral particles with masses

$$M_n = m\left(\frac{2g}{\pi + 2g}\right) n \qquad n = 1, 2, \ldots. \tag{3.5}$$

The $S$-matrix for two of these neutral particles is

$$S_{n,m} = \delta_{n-1,m} \left(\frac{\mathrm{i}e^\theta + 1}{e^\theta + \mathrm{i}}\right) \qquad \theta = \theta_n - \theta_m \quad m, n = 1, 2, \ldots \tag{3.6}$$

where $\theta_n$ is the rapidity of the $n$th neutral particle.

The charged fermions which appear in Korepin's solution at other values of $\beta$, become infinitely massive at $\beta^2 = 8\pi$ ($g = -\pi/2$) and, therefore, decouple. Notice that the neutral particles are not bound states, since (3.12) has no poles on the physical sheet [16].

Observe that the physical operator $\widehat{\sigma}$ is expressed in terms of $\widetilde{\psi}$ as $\widehat{\sigma}\widehat{\sigma} = \widetilde{\psi}^{*}_{(1)}\, \widetilde{\psi}_{(2)}$ and therefore the generation of the SC gap follows because $\langle 0|\, \widetilde{\psi}^{*}_{(1)}\, \widetilde{\psi}_{(2)}\, |0\rangle$ is non-vanishing in the massive Thirring model. The mass scale $m$ appearing in (3.5), in parcular, is fixed by $\langle\widehat{\sigma}\widehat{\sigma}\rangle$, which is proportional to $m$.

## 4. Conclusion

Using an appropriate bosonisation scheme, we have proved the equivalence of the O(2) symmetric Thirring model with dynamical generation of a superconducting gap to the exactly solved massive Thirring model at $g = -\pi/2$. The spectrum contains an infinite number of O(2) neutral particles.

It is interesting to note that an analogous equivalence was proved for the SU(2) chiral Gross–Neveu model [17]. Our model, however, is only equivalent to the O(2) sector of the chiral Gross–Neveu model since the physical field $\widehat{\chi}$ anticommutes with the additional SU(2) charge densities $\rho_1 = \overline{\psi}_1\gamma^0\psi_2 + \overline{\psi}_2\gamma^0\psi_1$ and $\rho_3 = \overline{\psi}_1\gamma^0\psi_1 - \overline{\psi}_2\gamma^0\psi_2$, as may be seen from (2.2), (2.4), (2.22) and (2.23). This fact implies that the physical states of the system bear zero $Q_1$ and $Q_3$ SU(2) charges. The same result may be obtained for the $\widetilde{\psi}$ field, by considering the densities $\rho_1$ and $\rho_3$ rewritten in terms of $\widetilde{\psi}$, accordingly. This observation, therefore, supports the view [18] that the Abelian bosonisation, when applied to the (non-Abelian) SU(2) Gross–Neveu model, just describes its O(2) sector, in contradistinction to the present (Abelian) case where the description is complete. It would be interesting to verify whether an exact solution may be found for $N > 2$.

We would like to remark that in a hypothetical application of this model to a realistic quasi-one-dimensional system of condensed matter, the dynamical generation of a superconducting gap, which we found, would become true superconductivity since, due to interchain coupling, the U(1) charge would no longer decouple from the physical sector.

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